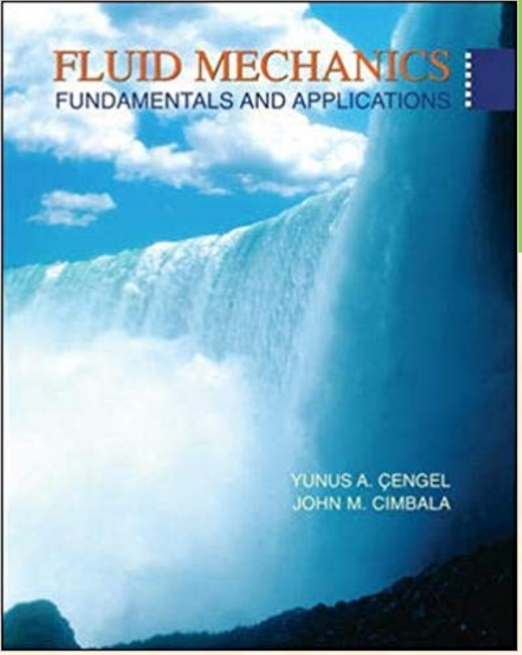

YUNUS A. ÇENGEL and JOHN M. CIMBALA, "Fluid Mechanics: Fundamentals and Applications",1st ed., McGraw-Hill,2006.

Course name

Fluid Mechanics I

Lecture-02- Chapter-04

Dımensional Analysis And Dynamic Similarity

Lecture slides by

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Outline

- ***Similarity Principles***
- ***Dynamic Simulated***
- ***Nondimensional Parameters***
- ***Simulation***
- ***Examples***
- ***Homeworks***

Similarity Principles

- In most experiments, to save time and money, tests are performed on a geometrically scaled **model**, rather than on the full-scale **prototype**.

➢ the principle of **similarity** is:

There are three necessary conditions for complete similarity between a model and a prototype.

- The first condition is **geometric similarity—** the model must be the same shape as the prototype, but may be scaled by some constant scale factor.
- The second condition is **kinematic similarity**, which means that the velocity at any point in the model flow must be proportional to the velocity at the corresponding point in the prototype flow (Fig. 7–16).
- The third and most restrictive similarity condition is that of **dynamic similarity**. Dynamic similarity is achieved when all *forces* in the model flow scale by a constant factor to corresponding forces in the prototype flow (*force-scale* equivalence).

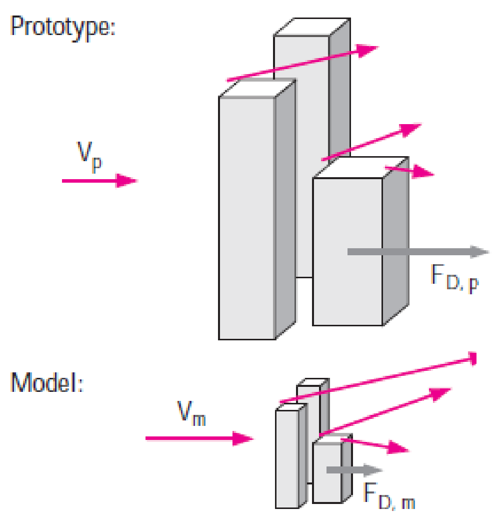
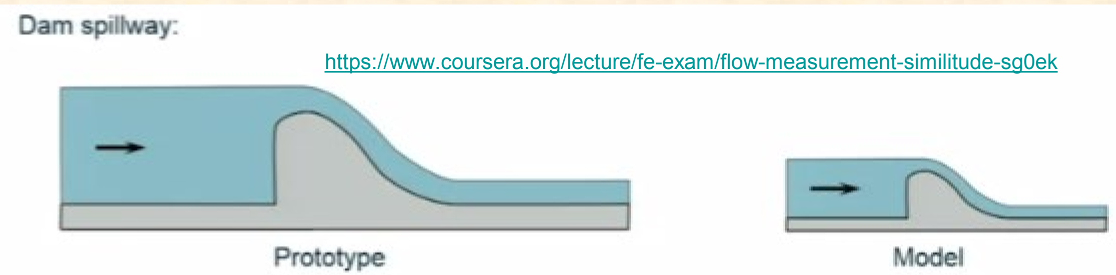


FIGURE 7–16
Kinematic similarity is achieved when, at all locations, the velocity in the model flow is proportional to that at corresponding locations in the prototype flow, and points in the same direction.



https://www.coursera.org/lecture/fe-exam/flow-measurement-similitude-sg0ek

3- Dynamic Simulated

Reynolds number:

1- Reynolds Number $Re = \dfrac{\text{inertia force}}{\text{viscous force}}$

inertia force $= m.a = \rho l^3 \dfrac{u}{t}$
$= \rho l^2 u \dfrac{l}{t} = \rho l^2 u^2$

viscous force $= l^2 \mu \dfrac{u}{l} = \mu l u$

$\therefore Re = \dfrac{\rho l^2 u^2}{\mu l u} \Rightarrow \boxed{Re = \dfrac{\rho l u}{\mu}}$

$$F_{in} = m.a = \rho \forall * \frac{u}{t} = \rho L^2 * u * \frac{L}{t} = \rho L^2 u^2$$

$$F_{vis} = \mu A \frac{u}{L} = \mu L^2 \frac{u}{L} = \mu L u$$

l = diameter of pipe = D إذا كان الجريان في الأنابيب

$l = L$ length of plate إذا كان الجريان على سطح مفتوح

يستخدم Re في حالة التشابه عند الجريان في الأنابيب، الجريان حول الأجسام المغمورة، الطبقات المتاخمة ... الخ

In the case of aerodynamic drag on an automobile, it turns out that if the flow is approximated as incompressible, there are only two π_s in the problem,

$$\Pi_1 = f(\Pi_2) \qquad \text{where} \qquad \Pi_1 = \frac{F_D}{\rho V^2 L^2} \qquad \text{and} \qquad \Pi_2 = \frac{\rho V L}{\mu} \qquad (7\text{–}13)$$

The procedure used to generate these Π's is discussed in Section 7–4. In Eq. 7–13, F_D is the magnitude of the aerodynamic drag on the car, ρ is the air density, V is the car's speed (or the speed of the air in the wind tunnel), L is the length of the car, and μ is the viscosity of the air. Π_1 is a nonstandard form of the drag coefficient, and Π_2 is the **Reynolds number**, Re. You will find that many problems in fluid mechanics involve a Reynolds number

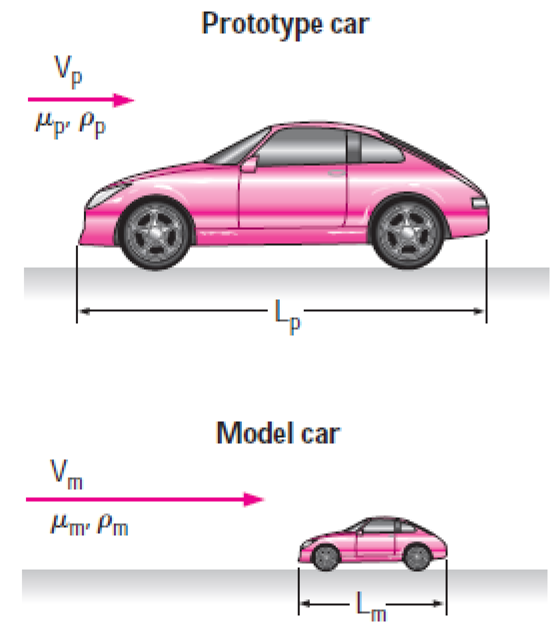


FIGURE 7–17
Geometric similarity between a prototype car of length L_p and a model car of length L_m.

3- Dynamic Simulated

The ***Froude number*** is an important dimensionless parameter in the study of open-channel flow

Froude number

$$Fr = \frac{V}{\sqrt{gD_m}}$$

where

V	The mean velocity
D_m	Hydraulic mean depth, A/B

Flow state

Critical	Fr = 1	or $V = \sqrt{gD_m}$
Supercritical	Fr>1	or $V > \sqrt{gD_m}$
Subcritical	Fr<1	or $V < \sqrt{gD_m}$

2 - Froud No. $Fr = \left(\frac{\text{Inertia force}}{\text{gravity force}}\right)^{1/2}$

gravity force $= m.a = \rho l^3 g$

inertia force $= \rho l^2 U^2$

$\therefore Fr = \left(\frac{\cancel{\rho} l^2 U^2}{\cancel{\rho} l^3 g}\right)^{1/2} \Rightarrow \boxed{Fr = \frac{U}{\sqrt{lg}}}$

يستخدم في حساب القفز العشوائي Hydraulic Jump

وكذلك في تصميم السدود dams — الخ

l: طول القناة اذا كانت المتناسبة

لإيجاد طول وكذلك العرض او العمق

- https://www.cram.com/flashcards/hydraulics-revision-channel-geometry-3514672

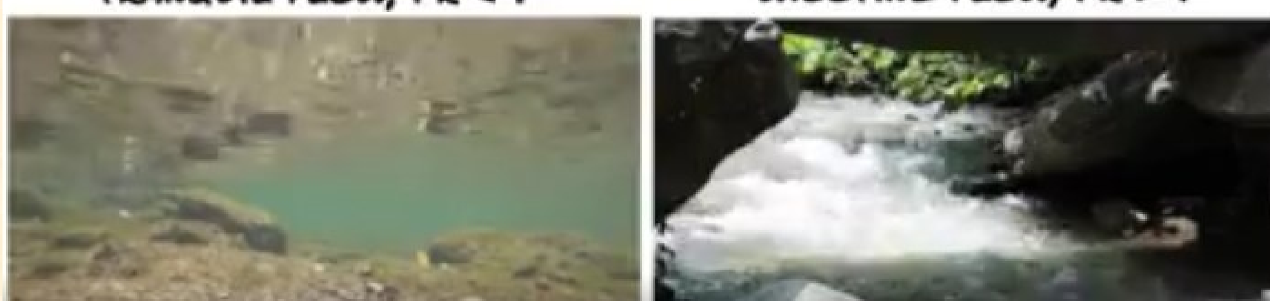
FROUDE NUMBER → DETERMINES IF THE FLOW IS TRANQUIL OR SHOOTING

FROUDE NUMBER: $Fr = \frac{U}{\sqrt{g \cdot L}}$

U → AVERAGE FLOW VELOCITY → INERTIAL FORCES

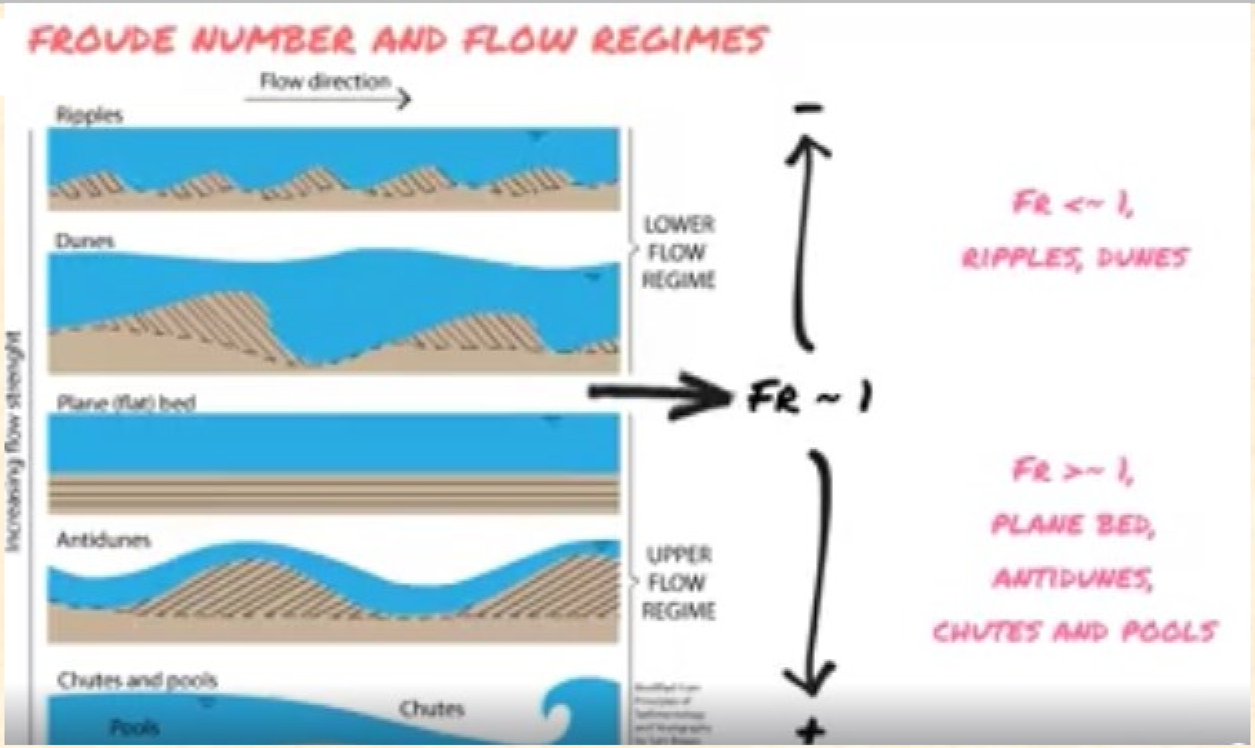
$\sqrt{g \cdot L}$ → WAVE VELOCITY IN SHALLOW WATER → GRAVITY FORCES

TRANQUIL FLOW, FR < 1 | SHOOTING FLOW, FR > 1



3- Dynamic Simulated

Froude number



$$F_r = \frac{V}{(gD)^{1/2}}$$

F_r = Froude number

V = average velocity of the liquid in a channel (feet/second or meters/second)

g = acceleration due to gravity (32.17 ft/sec^2 or 9.81 m/sec^2)

D = hydraulic depth

$$D = \frac{\text{cross-sectional area of flow}}{\text{top width}}$$

- https://www.pinterest.com/pin/616430886363031 95/

3- Dynamic Simulated

Euler number

3- Euler No. $\quad Er = \dfrac{\text{Pressure force}}{\text{Inertia force}}$

Inertia force $= \rho l^2 u^2$

$\therefore Er = \dfrac{P l^2}{\rho l^2 u^2} \Rightarrow \boxed{Er = \dfrac{\Delta P}{\rho u^2}}$

ΔP: Pressure drop in the flow

Euler's Number (Eu)

$$E_u = \sqrt{\frac{F_i}{F_p}} = \sqrt{\frac{\rho A V^2}{p \times A}} = \sqrt{\frac{V^2}{p/\rho}} = \frac{V}{\sqrt{p/\rho}}$$

Where F_p = Intensity of pressure × Area = p×A

And $F_i = \rho A V^2$

$(E_u)_{\text{model}} = (E_u)_{\text{prototype}}$

V_m = velocity of fluid in model,

p_m = pressure of fluid in model,

ρ_m = Density of fluid in model,

$V_p \quad p_p \quad \rho_p$ = Corresponding values in prototype, then

$(E_u)_{\text{model}} = (E_u)_{\text{prototype}}$

$$\frac{V_m}{\sqrt{p_m/\rho_m}} = \frac{V_p}{\sqrt{p_p/\rho_p}}$$

https://fluidfreak.wordpress.com/2014/05/03/reynolds-number/

3- Dynamic Simulated

Mach number

4- Mach No.

$$M = \left(\frac{\text{inertia force}}{\text{elastic force}}\right)^{1/2}$$

Elastic force $= E\ell^2 = \rho C^2 \ell^2$

C: speed of sound in the fluid

$= \sqrt{\gamma R T}$

$$\therefore M = \left(\frac{\rho \ell^2 u^2}{\rho c^2 \ell^2}\right)^{1/2} \Rightarrow M = \frac{u}{c} = \frac{\text{Speed of fluid}}{\text{Speed of sound}}$$

Mach No. effects are considered only when M exceeds 0.4 and when it is the only factors affecting the motion of flow.

a) subsonic $M < 1$ b) Sonic $M = 1$

c) supersonic M (1-7) d) hypersonic $M > 7$

8

Mach's Number (M)

Mach's number is defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force. Mathematically, it is defined as

$$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{F_i}{F_e}}$$

Where $F_i = \rho A V^2$

And F_e = Elastic force = Elastic stress × Area

$= K \times A = K \times L^2$ {K = elastic stress}

$$M = \sqrt{\frac{\rho A V^2}{K \times L^2}} = \sqrt{\frac{\rho \times L^2 \times V^2}{K \times L^2}} = \sqrt{\frac{V^2}{K/\rho}} = \frac{V}{\sqrt{K/\rho}}$$

But $\sqrt{\frac{K}{\rho}} = c = \text{velocity of sound in the fluid}$

$$M = \frac{V}{c}$$

Flow can be characterized using the **Mach number** as folllows:

(a) Ma ≤ 0.3: incompressible

(b) 0.3 < Ma < 1.0: compressible subsonic flow

(c) Ma ≥ 1.0: compressible supersonic flow

https://fluidfreak.wordpress.com/2014/05/03/reynolds-number/

3- Dynamic Simulated

Weber Number (We): The dimensionless parameter associated with surface tension effects is the Weber number, and it is defined as

We = $\rho V^2 L/\sigma$

5- Weber No

$$We = \frac{\text{Inertia force}}{\text{Surface tension force}}$$

$$We = \frac{\rho l^2 u^2}{\sigma l}$$

$$We = \frac{\rho l u^2}{\sigma}$$

σ = surface tension constant N/M

معامل الشد السطحي للسائل

$$(W_e)_{\text{model}} = (W_e)_{\text{prototype}}$$

If V_m = Velocity of fluid in model

σ_m = Surface tension force in model

ρ_m = Density of fluid in model

L_m = Length of surface in model,

And $V_p, \sigma_p, \rho_p, L_p$ = Corresponding values of fluid in prototype.

Then according to Weber law, we have

$$\frac{V_m}{\sqrt{\sigma_m/\rho_m L_m}} = \frac{V_p}{\sqrt{\sigma_p/\rho_p L_p}}$$

https://fluidfreak.wordpress.com/2014/05/03/reynolds-number/

Nondimensional Parameters

Dimensionless Parameters $f_i = r\frac{V^2}{l}$

- Reynolds Number $\quad Re = \frac{rVl}{m} \quad f_u = m\frac{V}{l^2}$
- Froude Number $\quad Fr = \frac{V}{\sqrt{gl}} \quad f_g = rg$
- Weber Number $\quad W = \frac{V^2 l\rho}{\sigma} \quad f_s = \frac{s}{l^2}$
- Mach Number $\quad M = \frac{V}{c} \quad f_{E_v} = \frac{rc^2}{l}$
- Pressure/Drag Coefficients $\quad C_p = \frac{-2(Dp + rgDz)}{rV^2} \quad C_d = \frac{2Drag}{\rho V^2 A}$
  - (dependent parameters that we measure experimentally)

Drag coefficient	$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$
Eckert number	$Ec = \frac{V^2}{c_p T}$	$\frac{\text{Kinetic energy}}{\text{Enthalpy}}$
Euler number	$Eu = \frac{\Delta P}{\rho V^2}\left(\text{sometimes } \frac{\Delta P}{\frac{1}{2}\rho V^2}\right)$	$\frac{\text{Pressure difference}}{\text{Dynamic pressure}}$
Fanning friction factor	$C_f = \frac{2\tau_w}{\rho V^2}$	$\frac{\text{Wall friction force}}{\text{Inertial force}}$
Fourier number	$Fo \text{ (sometimes } \tau) = \frac{\alpha t}{L^2}$	$\frac{\text{Physical time}}{\text{Thermal diffusion time}}$
Froude number	$Fr = \frac{V}{\sqrt{gL}}\left(\text{sometimes } \frac{V^2}{gL}\right)$	$\frac{\text{Inertial force}}{\text{Gravitational force}}$
Grashof number	$Gr = \frac{g\beta\|\Delta T\|L^3\rho^2}{\mu^2}$	$\frac{\text{Buoyancy force}}{\text{Viscous force}}$
Jakob number	$Ja = \frac{c_p(T - T_{sat})}{h_{fg}}$	$\frac{\text{Sensible energy}}{\text{Latent energy}}$
Knudsen number	$Kn = \frac{\lambda}{L}$	$\frac{\text{Mean free path length}}{\text{Characteristic length}}$

52

- https://www.slideserve.com/joy/dimensional-analysis-and-similitude
- https://www.slideshare.net/ADDISUDAGNEZEGEYE/fluid-mechanics-chapter-5-dimensional-analysis-and-similitude

Note: Just for briefing, you can See table 7–5 in the fluid mechanics book (pp.287-288) that shows some nondimensional parameters.

4- Simulation

4- Simulation

a) Geometrical similarity

Scale ratio, $\frac{l_m}{l_p} = \frac{t_m}{t_p} = \frac{D_m}{D_p}$ التشابه الهندسي

b) Kinematic Similarity

$V_r = \frac{V_p}{V_m}$

c) Dynamic Similarity

$$C_L = \left(\frac{F_L}{\frac{1}{2}\rho V^2 l^2}\right)_p = \left(\frac{F_L}{\frac{1}{2}\rho u^2 l}\right)_m$$

F_L: lift force C_L: lift coefficient

$$C_D = \left(\frac{F_D}{\frac{1}{2}\rho u^2 l^2}\right)_p = \left(\frac{F_D}{\frac{1}{2}\rho u^2 l^2}\right)_m$$

F_D: drag force or resistance force
دفع

$$C_f = \left(\frac{\tau}{\frac{1}{2}\rho u^2}\right)_p = \left(\frac{\tau}{\frac{1}{2}\rho u^2}\right)_m$$

τ: Shear Stress

C_f: Skins friction

Examples:

Example 1:

A model of Venturi meter linear dimensions one-fifth those of prototype. The prototype operates on water of $\nu = 1.007 \times 10^{-6}\ m/s^2$ and model of $0.3111 \times 10^{-6}\ m^2/s$. The throat diameter and velocity of the prototype is 600 mm and 6 m/s respectively. Calculate the flow rate of the model?

Sol. 1

Sol:-

$$(Re)_m = (Re)_p$$

$$\left(\frac{DV}{\nu}\right)_m = \left(\frac{DV}{\nu}\right)_p$$

$$\left(\frac{DV}{0.311 \times 10^{-6}}\right)_m = \left(\frac{D \times 6}{1.007 \times 10^{-6}}\right)_p$$

$$\therefore V = \frac{D_p}{D_m} \times \frac{6 \times 0.311}{1.007}$$

$$= 5 \times \frac{6 \times 0.311}{1.007} = 9.265\ m/s$$

Since

$$\frac{D_m}{D_p} = \frac{1}{5} = \frac{D_m}{0.6} \Rightarrow D_m = \frac{0.6}{5} \rightarrow D_m = 0.12\ m$$

$$Q_m = AV \Rightarrow Q_m = \frac{\pi}{4}(0.12)^2 \times 9.265$$

$$Q_m = 0.105\ m^3/s$$

Examples:

Example $_2$***: EXAMPLE 7–5*** **Similarity between Model and Prototype Cars**

The aerodynamic drag of a new sports car is to be predicted at a speed of 50.0 mi/h at an air temperature of 25°C. Automotive engineers build a onefifth scale model of the car to test in a wind tunnel. It is winter and the wind tunnel is located in an unheated building; the temperature of the wind tunnel air is only about 5°C. The wind tunnel has a moving belt to simulate the ground under the car, as in **Fig. 7–19**. Determine how fast the engineers should run the wind tunnel in order to achieve similarity between the model and the prototype.

Sol. $_2$

Properties For air at atmospheric pressure and at $T = 25°C$, $\rho = 1.184\ kg/m^3$ and $\mu = 1.849 \times 10^{-5}\ kg/m \cdot s$. Similarly, at $T = 5°C$, $\rho = 1.269\ kg/m^3$ and $\mu = 1.754 \times 10^{-5}\ kg/m \cdot s$.

Analysis Since there is only one independent Π in this problem, the similarity equation (Eq. 7–12) holds if $\Pi_{2,m} = \Pi_{2,p}$, where Π_2 is given by Eq. 7–13, and we call it the Reynolds number. Thus, we write

$$\Pi_{2,m} = Re_m = \frac{\rho_m V_m L_m}{\mu_m} = \Pi_{2,p} = Re_p = \frac{\rho_p V_p L_p}{\mu_p}$$

which can be solved for the unknown wind tunnel speed for the model tests, V_m,

$$V_m = V_p\left(\frac{\mu_m}{\mu_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{L_p}{L_m}\right)$$

$$= (50.0\ mi/h)\left(\frac{1.754 \times 10^{-5}\ kg/m \cdot s}{1.849 \times 10^{-5}\ kg/m \cdot s}\right)\left(\frac{1.184\ kg/m^3}{1.269\ kg/m^3}\right)(5) = 221\ mi/h$$

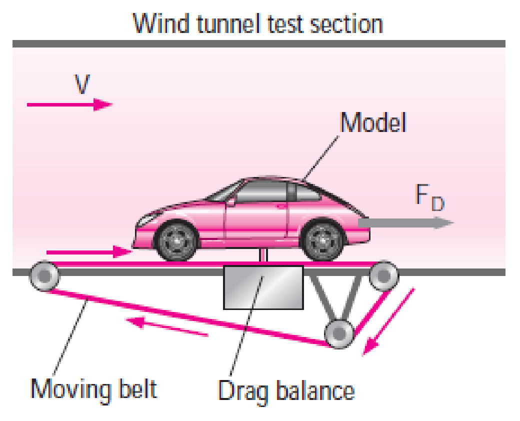


FIGURE 7–19
A drag balance is a device used in a wind tunnel to measure the aerodynamic drag of a body. When testing automobile models, a moving belt is often added to the floor of the wind tunnel to simulate the moving ground (from the car's frame of reference).

Examples:

***Example $_3$*: *EXAMPLE 7–6* Prediction of Aerodynamic Drag Force on the Prototype Car**

This example is a follow-up to Example 7–5. Suppose the engineers run the wind tunnel at 221 mi/h to achieve similarity between the model and the prototype. The aerodynamic drag force on the model car is measured with a **drag balance** (Fig. 7–19). Several drag readings are recorded, and the average drag force on the model is 21.2 lbf. Predict the aerodynamic drag force on the prototype (at 50 mi/h and 25°C).

Sol. $_3$

SOLUTION Because of similarity, the model results can be scaled up to predict the aerodynamic drag force on the prototype.

Analysis The similarity equation (Eq. 7–12) shows that since $\Pi_{2,m} = \Pi_{2,p}$, $\Pi_{1,m} = \Pi_{1,p}$, where Π_1 is given for this problem by Eq. 7–13. Thus, we write

$$\Pi_{1,m} = \frac{F_{D,m}}{\rho_m V_m^2 L_m^2} = \Pi_{1,p} = \frac{F_{D,p}}{\rho_p V_p^2 L_p^2}$$

$$\Pi_{1,m} = \Pi_{1,p} \qquad (7\text{–}12)$$

which can be solved for the unknown aerodynamic drag force on the prototype car, $F_{D,p}$,

$$F_{D,p} = F_{D,m}\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2$$

$$= (21.2\ \text{lbf})\left(\frac{1.184\ \text{kg/m}^3}{1.269\ \text{kg/m}^3}\right)\left(\frac{50.0\ \text{mi/h}}{221\ \text{mi/h}}\right)^2(5)^2 = 25.3\ \text{lbf}$$

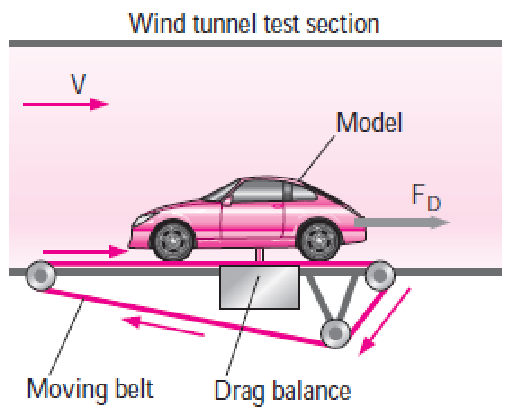


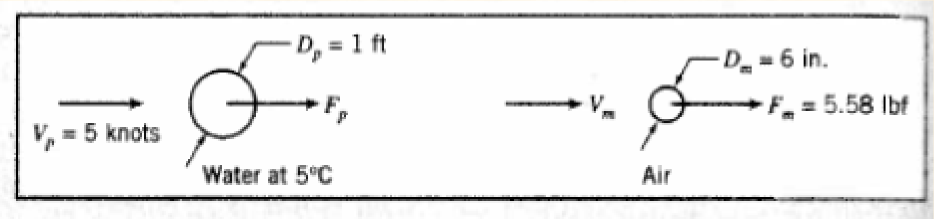
FIGURE 7–19
A drag balance is a device used in a wind tunnel to measure the aerodynamic drag of a body. When testing automobile models, a moving belt is often added to the floor of the wind tunnel to simulate the moving ground (from the car's frame of reference).

Homeworks:

Hw1: A prototype ship is 35 m long and designed to cruise at 11 m/s (about 21 kn). Its drag is to be simulated by a 1 - m-long model pulled in a tow tank. For Froude scaling find (a) the tow speed, (b) the ratio of prototype to model drag, and (c) the ratio of prototype to model power.

Hw2: A prototype ocean platform piling is expected to encounter currents of 150 cm/s and waves of 12s period and 3m height. If a one-fifteenth-scale model is tested in a wave channel, what current speed, wave period, and wave height should be encountered by the model?

Hw3: The drag of a sonar transducer is to be predicted based on wind tunnel data. The prototype, a 1ft diameter sphere, is to be towed at 5 knots (nautical miles per hour) in seawater at 5° C. The model is 6 in. in diameter. Determine the required test speed in air. If the drag of the model at these test conditions is 5.58 lb_f, estimate the drag of the prototype. Is this test valid? Why or why not? (20 pts)



- Note:
  - Solve all three Homeworks and sending me the answering next week on Sunday 5 April 2024.
  - Read examples 7-10 & 7-11 in the fluid mechanics book (pp.299-302)
- I hope everything is clear for all students

Good luck